Quiz 5 Solutions!

1. Consider the following example and answer the ensuing questions.

2	2	1	1
B	C	A	D
A	A	D	B
C	B	B	A
D	D	C	C
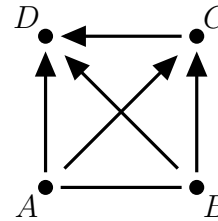
(a) What is the Smith set? (1 pt) $S = \{\, A, B \,\}$

(b) Who is/are the winners for plurality? (1 pt) $W_P = \{\, B, C \,\}$

(c) Does this preference schedule serve as a counterexample to the following claim:

Claim: Plurality is Smith fair.

(2 pts) <u>Circle one</u>   Yes   No

Explain briefly:

To be Smith fair, the winners for plurality must be Smith candidates in every election ever. In this example, C is a plurality winner but NOT a Smith winner, so plurality is NOT Smith fair.

(d) Who wins by Runoff in this example? (1 pt) $W_R = \{\, B \,\}$

(e) Is B a weak spoiler for Runoff? (1 pt)   Circle one: Yes   No
    $B \in S$ so B cannot be a weak spoiler.

(f) Is C a weak spoiler for Runoff? (1 pt)   Circle one: Yes   No
    When C is disqualified, the new winner set for runoff is $\{A, B\} \neq \{B\}$.

(g) Is D a weak spoiler for Runoff? (1 pt)   Circle one: Yes   No
    When D is disqualified the winner set for Runoff is the same.

2. Prove: If C is a Condorcet candidate then the Smith set is $S = \{C\}$. <u>Write the definitions</u>. (2 pts)

   Proof.   • The Smith set is the smallest dominating set.
     • Since C wins every head-to-head battle, $\{C\}$ is a dominating set
     • and $\{C\}$ is as small as possible. So $S = \{C\}$.