

Quiz 13 Solutions

Two seniors are taking a group exam which has three problems. The problems are of “equal size”, in the sense that each question should take one hour for one person. The wise seniors decide that to maximize their success, they should divide the three questions between them according to their individual preferences.

1. Quantifying value. (4 pts)

- (a) For every two minutes that senior *A* works on Question 1, she would like to spend three minutes on Question 2 and three minutes on Question 3. If x is the portion of her time that *A* works on Question 1, make an equation in x which describes *A*'s ideal allocation of her time across the three questions as fractions of a whole: (note- there need not be any units associated with x , because x is a ratio!)

• $2x + 3x + 3x = 1$
 or
 • $x + \frac{3}{2}x + \frac{3}{2}x = 1$

both are good

- (b) Describe *A*'s value for the three components of the exam as portions of the hour (what is *A*'s ideal hour?) by filling out the table below:

	Q1	Q2	Q3
A	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$
B	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$

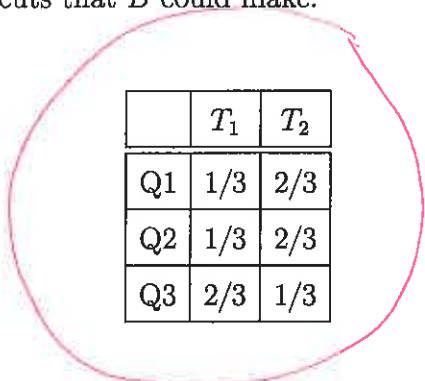
(give same solutions as ratios)

Note that *B*'s values have already been filled out! You do NOT need to compute these for *B* going forward.

2. Cuts fair to *B* in “I cut you choose” (3 pts)

- (a) Suppose now *B* is going to “make a cut”. In this context, that means *B* is going to divide each of the three questions across the two players, in fractions of an hour (time needed for one question). Below are three cuts that *B* could make:

	S_1	S_2
Q1	0	1
Q2	$\frac{1}{3}$	$\frac{2}{3}$
Q3	1	0



	T_1	T_2
Q1	$\frac{1}{3}$	$\frac{2}{3}$
Q2	$\frac{1}{3}$	$\frac{2}{3}$
Q3	$\frac{2}{3}$	$\frac{1}{3}$

	R_1	R_2
Q1	$\frac{3}{4}$	$\frac{1}{4}$
Q2	$\frac{1}{4}$	$\frac{3}{4}$
Q3	$\frac{3}{4}$	$\frac{1}{4}$

Circle which cuts *B* would make to guarantee getting *B*'s fair share, no matter what *A* picks!

3. Playing "I cut, you choose" with A and B .

In the end, B decides to make this cut:

	P_1	P_2
Q1	$1/4$	$3/4$
Q2	0	1
Q3	$5/6$	$1/6$

- (a) Make the envy-table, assuming the Questions are divided into slices P_1 and P_2 as on this page. In other words: what does A think each slice is worth in A 's eyes, and what does B think each slice is worth in B 's eyes? (2 pts)

$$A \text{ val } P_1 = \frac{1}{4} \times \frac{1}{4} + \frac{3}{8} \times \frac{5}{6} = \frac{1}{16} + \frac{5}{16} = \frac{6}{16} = \frac{3}{8}$$

	P_1	P_2
A	$3/8$	$5/8$
B	$1/2$	$1/2$

- (b) Which slice would A pick to get A 's fair share? (1 pt) Circle One: P_1 P_2

- (c) Give the slice you circled in Question 3b to A and give the other slice to B .

Circle all properties which are true for this division: (2 pts)

fair envy-free equitable

- (d) There are a lot of cuts that B could have made in I cut you choose to guarantee getting B 's fair share. For some arbitrary cut, Let x be the portion of Question 1, y be the portion of Question 2, and z be the portion of Question 3 in the first slice.

- i. Make an equation in x, y and z which describes B guaranteeing getting B 's fair share with this cut. (2 pts)

$$\frac{1}{3}x + \frac{1}{6}y + \frac{1}{2}z = \frac{1}{2}$$

- ii. If B puts none of Question 2 and $1/3$ of Question 3 in the first slice, what portion of Question 1 does B need to put in that slice to guarantee getting B 's fair share? (1 pt)

$x = 1$ (all of Question 1)