# Quiz 11 Solutions

1. The psych students $A, B$, and $C$ submit bids $\boxed{a = 6 \qquad b = 4 \qquad c = 5}$ for their advisors outdated Weschler IQ test.

   (a) Suppose $A$ is the winning bidder. Find an equitable compensation arrangement. (3 pts)

$$q = \frac{w}{S} = \frac{6}{6+4+5} = \frac{6}{15} = \frac{2}{5}$$
$$x_B = qb = \frac{2}{5} \times 4 = \frac{8}{5}$$
$$x_C = qc = \frac{2}{5} \times 5 = 2$$

   (b) Now suppose $C$ is the winning bidder. The payouts are

$$x_A = 2 \qquad\qquad x_B = 1$$

   What does

   i. $A$ think $A$ gets? 2 (1 pt)

   ii. $A$ think $C$ gets? 6 - 2-1 $= 3$ (1 pt)

   iii. $C$ think $C$ gets? 5-2-1 $= 2$ (1 pt)

   iv. Circle which players have envy: $\boxed{\text{A}}$ $\boxed{\text{B}}$ $C$ (1 pt)

2. Now $A$ and $B$ submits bids $\boxed{a, b}$ which are some positive real numbers, NOT the number from Question 1. If $A$ is the winning bidder, what is the equitable payout to $B$ that $A$ should make? Show work. (3 pts)

   bids: $a$, $b$ unknown real numbers. $A$ wins. Find $q$, $x_B$

$$x_B = \frac{ab}{a+b}$$

   See the solution to Question 14.5(a) in the book.

3. **Extra credit** (+1 pt) Prove that the payout you found in Question 2 is fair to $B$ if and only if $A$ is a highest bidder. Write on the back!

   *Proof.* Fair to $B$ means $x_B = \frac{ab}{a+b} \geqslant \frac{b}{2}$. Rearranging this equation gives us what we want:

$$\frac{ab}{a+b} \geqslant \frac{b}{2}$$
$$\iff \frac{a\not{b}}{a+b} \geqslant \frac{\not{b}}{2}$$
$$\iff \frac{a}{a+b} \geqslant \frac{1}{2}$$
$$\iff 2a \geqslant a + b$$
$$\iff 2a - a \geqslant b \iff a \geqslant b$$

   which means $A$ is a highest bidder. □