Quiz 9

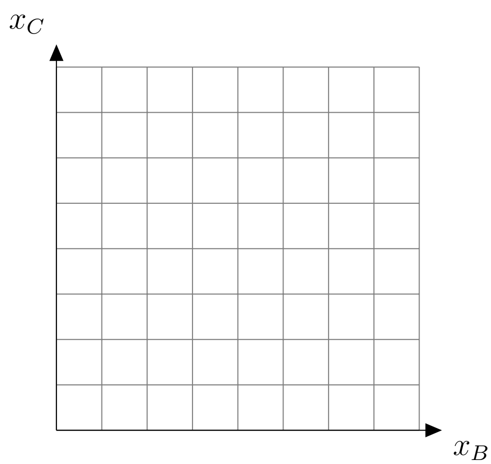
1. **Throughout this question, A and C have bids**

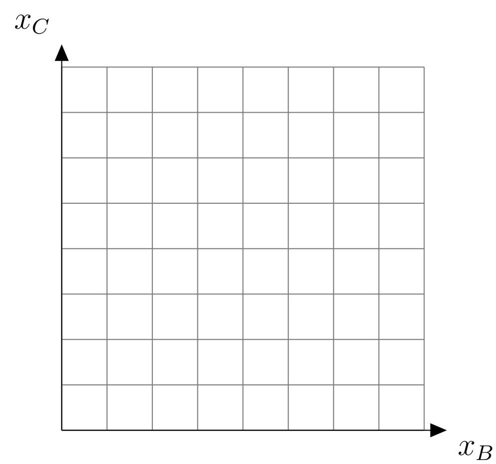
$$a = 9 \qquad c = 6$$

and A is the winning bidder. B's bid will be different depending on certain parts of the question.

(a) In the (x_B, x_C)-plane below on the left, describe all compensation arrangements fair to C.

(b) In the (x_B, x_C)-plane below on the right, describe all compensation arrangements fair to A.





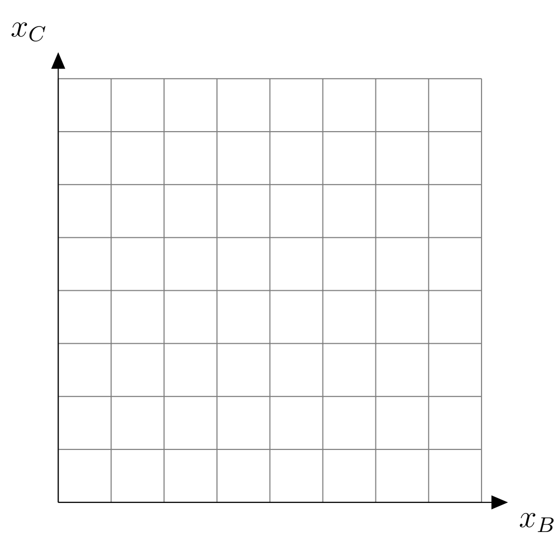
(c) Draw the line representing C getting C's fair share and the line representing A getting A's fair share on the same (x_B, x_C)-plane below. Plot the point $(3, 3)$ in the plane.

i. On this same plane, draw a line representing B getting B's fair share if $b = 6$. Label this line $b = 6$.

ii. On this same plane, draw a line representing B getting B's fair share if $b = 9$. Label this line $b = 9$.

iii. On this same plane, draw a line representing B getting B's fair share if $b = 12$. Label this line $b = 12$.

iv. Graph the line $x_C = x_B$, ie- $y = x$, and label it.



(d) For which values of b is the point $(3,3)$ in the fairness triangle? Circle:

$b = 6$ $\qquad$ $b = 9$ $\qquad$ $b = 12$

(e) Applying Proposition 13.14 part 3., for which values of b does the point $(3,3)$ correspond to an envy-free compensation arrangement? Circle:

$b = 6$ $\qquad$ $b = 9$ $\qquad$ $b = 12$

(f) Are there any values of b for which NO envy-free compensation arrangement is possible? If yes, circle them:

$b = 6$ $\qquad$ $b = 9$ $\qquad$ $b = 12$

(g) True or False:

In this example, an envy-free arrangement is possible if and only if $b \leqslant 9$. T F

(h) **Prove for <u>any</u> bid values**: The winning bidder, A, is a highest bidder if and only if $x_B = x_C = \frac{a}{3}$ is a fair compensation arrangement.

Partner: ______________________________ Your Name: ______________________________

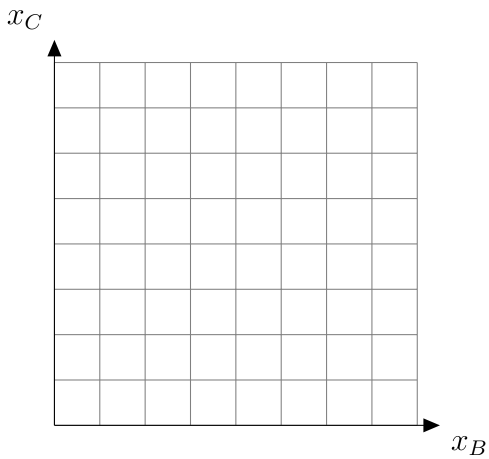
2. **Throughout this question, B and C have bids**

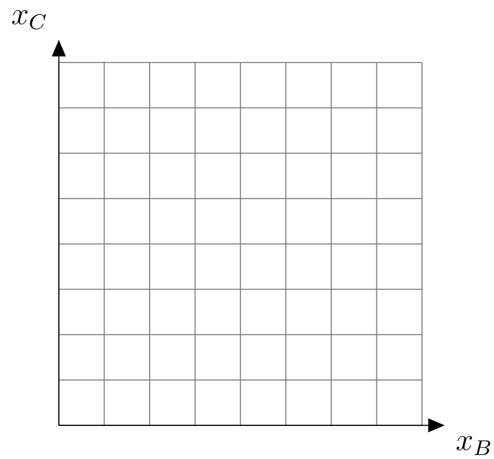
$$b = 12 \qquad c = 6$$

and A is the winning bidder. A's bid will be different depending on certain parts of the question.

(a) In the (x_B, x_C)-plane below on the left, describe all compensation arrangements fair to B.

(b) In the (x_B, x_C)-plane below on the right, describe all compensation arrangements fair to BOTH B and C.





(c) Draw the line representing B getting B's fair share and the line representing C getting C's fair share on the same (x_B, x_C)-plane on the next page.

i. Plot the point which represents both B and C getting exactly their fair shares. What is this point?

$$P = (\underline{\hspace{3cm}})$$

ii. On this same plane, draw a line representing A getting A's fair share if $a = 12$. Label this line $a = 12$.

iii. On this same plane, draw a line representing A getting A's fair share if $a = 9$. Label this line $a = 9$.

iv. On this same plane, draw a line representing A getting A's fair share if $a = 6$. Label this line $a = 6$.

(d) For which values of a was a fair arrangement possible? Circle:

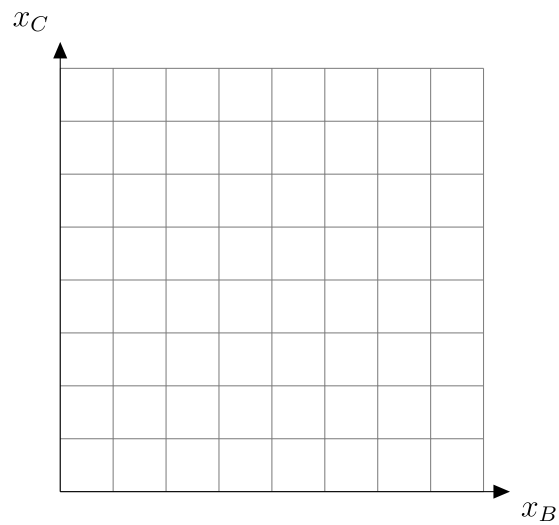
$$a = 6 \qquad\qquad a = 9 \qquad\qquad a = 12$$

(e) What is the average bid when

$a = 6$?

$a = 9$?

$a = 12$?

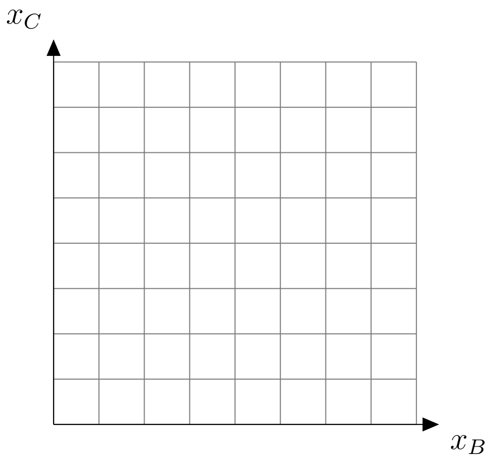


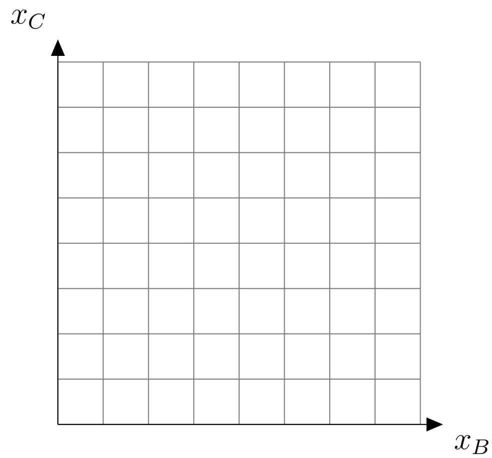
(f) Prove: for any real numbers a, b, and c,

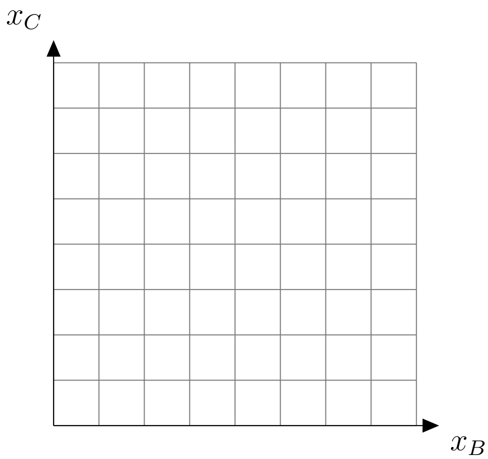
$$a = \frac{a+b+c}{3} \iff a = \frac{b+c}{2}$$

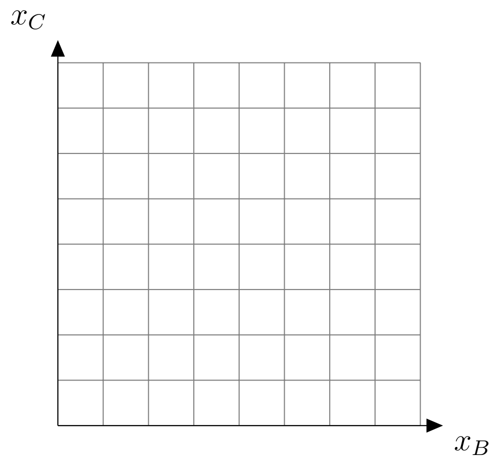
(g) Prove: for any real numbers a, b, and c,
If A, the winning bidder, is an average bidder, then $(b/3, c/3)$ is a point on the line representing A getting A's fair share. Conclude that the fairness triangle is only equal to one point.

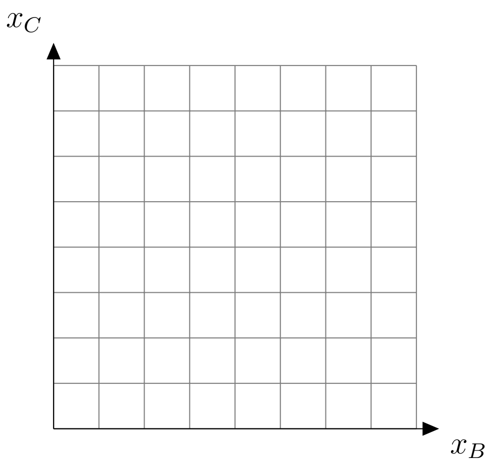
Partner: ______________________________ Your Name: ______________________________

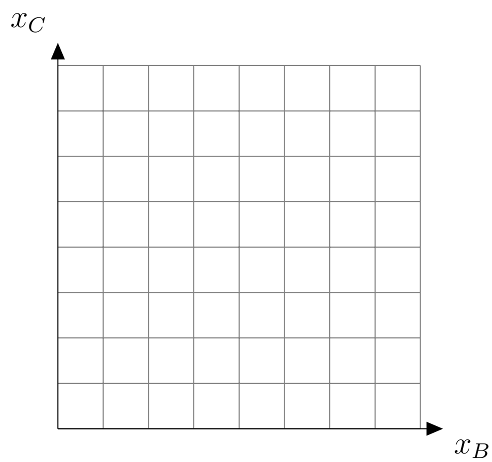

x_C
x_B


x_C
x_B


x_C
x_B


x_C
x_B


x_C
x_B


x_C
x_B