Exploring properties of compensation arrangements with Mathematica

Instructions: Go to the Compensation Applet at the bottom of the course webpage, click on the image, and download the Wolfram CDF player. The Wolfram CDF application is free for Tufts students. You should now be able to view the interactive compensation applet, unless you are using Chrome, in which case you will need to download the link to the .cdf document and open with the Wolfram CDF player application.

Throughout this worksheet, we assume there are three bidders and A is the winning bidder. Then a compensation arrangement is determined by A's payouts to B and C, which we represent as pairs of points in the (x_B, x_C)-plane.

1. In the menu on the left: Expand Values of the Object and set A's bid to 60 by pressing the plus sign to the right of the slider and typing "60" in the white box.
   Similarly, set B's bid to 45, and C's bid to 30. Clicking on the graph will move the dynamic point. You can drop down the point location for it's coordinates.

   (a) Click on the point which represents B and C getting exactly their fair shares. Use the Point Location to find the coordinates (drop down on the left). What is the compensation arrangement associated to this point? Is this compensation arrangement fair to A?

   (b) Click on the blue line. Use the Point Location to find the coordinates (drop down on the left). What is the compensation arrangement associated to this point? What is A's payout?

   (c) Click on a different point on the blue line. What is A's payout now? If you click on another point on the blue line, what do you expect A's payout to be?

   (d) Click on a point above the blue line. Is this compensation arrangement fair to A?

2. Set B's bid to 12 and C's bid to 24. In this exercise, we will vary A and see what happens.

   (a) Set $A = 27$. Which bidder(s) is/are a highest bidder(s)? Which bidders are at least average? Is a fair compensation arrangement possible? If yes, named one compensation arrangement which is fair and one which is not (using the graph).

   (b) Slide A's bid to 21. Which bidder(s) is/are a highest bidder(s)? Which bidders are at least average? Is a fair compensation arrangement possible?

   (c) Slide A's bid to the left until the blue, purple, and orange lines all intersect. What is the value of A's bid which makes this happen?

   (d) Let m be the value you found in the last question. Prove for this example that $a = m$ if and only if a is an average bidder, using b and c as given in this problem but let A vary. (Hint: set up the equation which represents a being an average bidder and solve for x).

   (e) If you slide A's bid any further to the left, will a fair compensation arrangement be possible?

3. Set A's bid to 24 and B's bid to 12. Activate the Equal Compensation Amounts option.

   (a) Applying Proposition 13.14: If C's bid is 18, is an envy-free compensation arrangement possible?

(b) Click on the intersect point between the green line and the blue line. What is the compensation arrangement associated to this point? Is it fair? Is it envy-free?

(c) Increase C's bid until the yellow, blue, and green lines are intersect. What is the intersection point? Note that this point represents the only envy-free compensation arrangement.

(d) What is the value of C's bid which makes the yellow, blue, and green lines intersect? What is the value of A's bid? (Remember that A is the winning bidder.)

(e) If you increase C's bid any further, is an envy-free compensation arrangement possible?

4. Set B's bid to 9 and C's bid to 15. Activate the Equitable Arrangement. The black point is the point associated to the equitable compensation arrangement $x_A = qa, x_B = qb, x_C = qc$ where $q = w/S$.

5. Set A's bid to 15 and slide A's bid to the right. What happens to the equitable arrangement? Is it fair?

6. Decrease A's bid by sliding it to the left. Describe the range of values for A's bid such that the equitable arrangement is fair.

7. Let's compare values of q as we vary A.

(a) What is q when $a = 9$?

(b) What is q when A is an average bidder? (it is printed below the Equitable Arrangement option in the menu on the left.)

(c) What is q when $a = 20$?

(d) What is q when $a = 24$?

(e) Circle the word that fills in the blank:

As we increase A's bid, q _____ .

Circle one: increases decreases

In particular, this means as we increase A's bid, players are getting a _____ proportion of their bid in the equitable compensation arrangement.

Circle one: larger smaller

8. Try arranging A, B, and C's bids to make the equitable arrangement sit on the green line.

(a) What must be true of B's and C's bids to make this happen?

(b) Once you have the equitable arrangement on the green line, slide A's bid. What is happening to the black point? Why?

(c) Can you describe when the equitable arrangement will be envy-free?