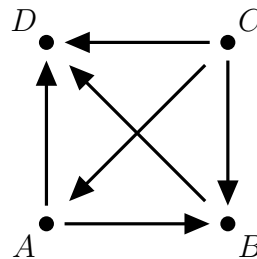


Homework 8: Chapters 5 and 6

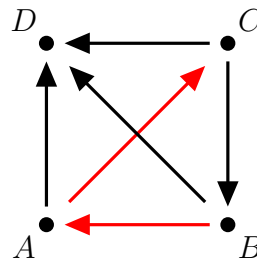
1. 5.1 (Solution)
2. 6.1, 6.3 (Solutions)
3. Exercise (5.4) is to show that the Silly method is not Smith fair. Here's the example from class:

2	1	1	1
<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>
<i>C</i>	<i>B</i>	<i>C</i>	<i>A</i>
<i>A</i>	<i>C</i>	<i>A</i>	<i>B</i>
<i>B</i>	<i>D</i>	<i>D</i>	<i>D</i>



And here is one way to tweak the example from class:

2	1	1	1
<i>D</i>	<i>A</i>	<i>C</i>	<i>C</i>
<i>B</i>	<i>C</i>	<i>B</i>	<i>A</i>
<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>
<i>C</i>	<i>D</i>	<i>D</i>	<i>D</i>

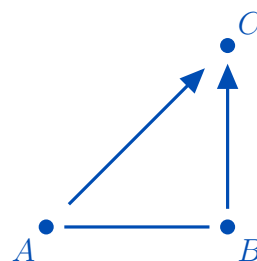


Show that Silly method is not Smith fair using this example.

Solution. We want to show that there is a winner for the Silly method who is not in the Smith set. As we saw in class, there needs to not be a Condorcet candidate. With the adjustment, the Smith set is $S = \{A, B, C\}$ (check). Since there is no Condorcet candidate, the Silly method chooses the candidates with the plurality vote as the winners - these candidates are D and C . But D is not a Smith candidate, so the Silly method is NOT Smith fair.

4. Exercise (5.7) is to show that Silly Borda a.k.a. a posteriori Smith fair Borda count is not a priori Smith fair. Check that this example is a counterexample:

2	1	1
<i>B</i>	<i>A</i>	<i>A</i>
<i>A</i>	<i>B</i>	<i>C</i>
<i>C</i>	<i>C</i>	<i>B</i>



Solution. The Smith set is $S = \{A, B\}$. This set is dominating and could not be any smaller because A and B tie. Since Silly Borda only cares about Borda scores in the Smith set, we only need to compute those two scores:

$$\mathcal{B}(A) = 3 \times 2 + 2 \times 2 = 10$$

$$\mathcal{B}(B) = 3 \times 2 + 2 \times 1 + 1 \times 1 = 9$$

So the winner set for Silly Borda is $W = \{A\}$.

Now remove $C \notin S$. The new preference schedule is:

2	1	1	=	2	2
B	A	A		B	A
A	B	B		A	B

The Smith set is still $\{A, B\}$ and clearly A and B will have the same Borda scores, which we can compute to be:

$$\mathcal{B}'(A) = 2 \times 2 + 2 \times 1 = 6$$

$$\mathcal{B}'(B) = 2 \times 2 + 2 \times 1 = 6$$

So the new winner set for Silly Borda is $\{A, B\}$ and C is a weak spoiler for Silly Borda.