# Homework 7: Chapters 4 and 5 - Solutions

Exercises:

1. 4.3, 4.5 (Solutions), 4.6 (Solution).

2. 5.3 - prove that Smith fair methods are Condorcet fair directly by definition, not by the sneaky thing from class. (Solution). (on a quiz, either a nicely written version of the proof from class or the direct proof would be acceptable.)

3. 5.2 (Solution)

4. Finish our computation for the example from class:

1	2	1	1	2	2
D	D	D	A	B	C
A	B	C	B	C	A
B	C	A	C	A	B
C	A	B	D	D	D
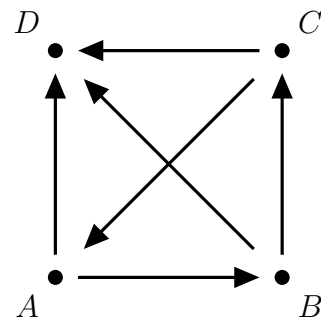
First compute S to verify that D is the only one with the power to be a weak spoiler. Make sure you understand what that means. Then check if D is a weak spoiler for:

(a) Plurality: First, the winner set for plurality is $W_P = \{D\}$. Since $D \notin S$, when D is disqualified W_P changes no matter what! In particular, $W_P' = \{B\}$. So YES, D is a weak spoiler for plurality.

(b) Runoff The candidates with the most and second-most first place votes continue in the runoff election, which is determined by a plurality vote. Since there is a tie for second-most first place votes, D, B, and C all enter the runoff election, which D wins. Again the original winner set with D for runoff is $W_R = \{D\}$, and this changes when D is disqualied. Thus D is a weak spoiler for runoff.

(c) Instant Runoff/Elimination By the elimination method, A is eliminated, then C, and B beats D in h2h, so $W_E = \{B\}$. Now disqualify D. The new preference schedule, for convenience:

1	2	1	1	2	2
A	B	C	A	B	C
B	C	A	B	C	A
C	A	B	C	A	B

Now A is disqualified first, and B beats C in h2h so $W_E' = \{B\}$. The winner set is unchanged and D is NOT a weak spoiler.

(d) Pairwise Comparison: $W_{PC} = \{A, B, C\}$ is the same, with or without D.

(e) Borda Count: with D,

$$\mathcal{B}(A) = 4 + 3 \times 3 + 2 \times 3 + 1 \times 2 = 21$$
$$\mathcal{B}(B) = 4 \times 2 + 3 \times 3 + 2 \times 3 + 1 = 24$$
$$\mathcal{B}(C) = 4 \times 2 + 3 \times 3 + 2 \times 3 + 1 = 24$$
$$\mathcal{B}(D) = 4 \times 4 + 5 = 21$$

So $W_B = \{B, C\}$. Now with D disqualified,

$$\mathcal{B}(A) = 3 \times 2 + 2 \times 3 + 4 = 16$$
$$\mathcal{B}(B) = 3 \times 4 + 2 \times 2 + 3 = 19$$
$$\mathcal{B}(C) = 3 \times 3 + 2 \times 4 + 2 = 19$$

Then $W_B' = W_B$ and, disappointingly, D is not a weak spoiler for Borda count.

Next Quiz: There will be a proof or counterexample type question on the next quiz! It will be one of the questions from this homework or the Chapter 4 homework.