Homework 6: Chapter 4 - Solutions!

Exercises:

1. 4.1, 4.2 (Book solutions).

2. Explain what happens to the Smith set when there are Condorcet candidates. In other words, complete Exercise (4.4). (Book solutions).

3. It was remarked in class that the Smith method cannot have losing spoilers. Woohoo! But does the Smith method satisfy the Retroactive Disqualification Criterion? Your assignment:

   Prove or give a counterexample: The Smith method never has winning spoilers.

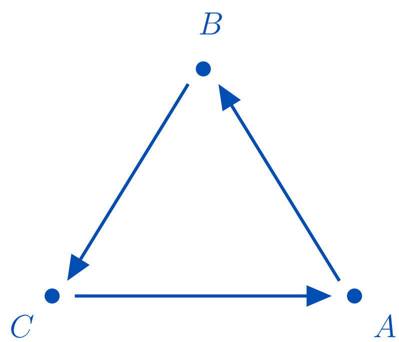
   (Just a remark) It is not uncommon to see a lot of candidates in the Smith set. There are a lot of winners sharing space - which means lots of potential for winning spoilers. This is why we might start to think that the Smith method can have winning spoilers. But even without this intuition, the best approach to this question is to try out examples.

   So I encourage you to play with some examples we already have from class and check for winning spoilers for the Smith method. And I'll supply an example below, so if you aren't sure how one would come up with a counterexample, you have at least one example of a winning spoiler and could come up with others.

   Solution. This claim is false. The Smith method can have winning spoilers.

   Let S be the Smith set, which is the winner set for the Smith method. We want an example election, meaning a *preference schedule*, such that S is big, and one candidate $X \in S$ changes the Smith set in bad ways when disqualified. "Bad ways" means S', the new Smith set without X, is such that $S' \neq S - \{X\}$.

   Below is a Comparison graph that does what I want. The Smith set is $S = \{A, B, C\}$. *Every* candidate is a winning spoiler for the same reason. For example, if we remove A, then the new Smith set is $S' = \{B\}$, but it should have been $S - \{A\} = \{B, C\}$. This makes A a winning spoiler.



To have a counterexample to the exercise, I need a *preference schedule* which has this Comparison graph. Here's one (of many possible preference schedules):

1	1	1
C	B	A
A	C	B
B	A	C

□

4. Is the Smith method majority fair? Explain in one sentence!

Proof. Yes - here are two one-liner proofs.

(a) We've seen in class that the Smith method is Condorcet fair and that all Condorcet fair winner selection methods are also majority fair.

(b) If there is a majority candidate M, they are a Condorcet candidate, and by Exercise 4.4 they are in the Smith set which is just $S = \{M\}$.

□