Homework 4: Chapter 2 Solutions!

Solutions to exercises from the book.
Solutions to optional problems:

1. (optional) Recall the EXTREMELY FALSE Claim F from class:

   Claim F: Every Condorcet candidate is a majority candidate.

   You've seen counterexamples to this claim. Construct a counterexample with n and N as small as possible.

   Solution. When there are $n = 2$ candidates, Condorcet candidates and majority candidates coincide. So we'll need at least $n = 3$. Also, to beat everyone in a head-to-head without the majority, 2 votes wouldn't suffice. Would 3? Yes, here's an example:

1	1	1
A	B	C
B	A	A
C	C	B

   Check that there is a Condorcet candidate in this example who is not a majority candidate. If n or N is less than 3, a counterexample to Claim F would be impossible. □

2. (optional) Practice proving these claims:

   (a) If $n \leqslant 3$, then elimination and runoff choose the same winners.

   (b) If $n = 2$, then runoff and pairwise comparison choose the same winners as plurality.

   (c) There can only be at most one majority cand. in an election.

   (d) There can only be at most one Condorcet cand. in an election.

   Solutions. (a) Suppose $n = 3$ and there are no ties for # 1st pl votes. Then only one candidate can have the least first place votes, so they are the *only candidate* cut in the runoff election. They were cut in the first round of elimination by definition of the elimination WSM.

   In other words: when $n = 3$, both WSM's reduce to a head-to-head battle between candidates with the most first place votes. If there was a tie at the beginning somewhere, you could look at it case by case and realize that the winners are the same.

   (b) You can check by definition also. Here is an alternate proof. When $n = 2$, either there is a tie and both candidates win by all methods, or there is one candidate has the majority. Since runoff, pairwise comparison, and plurality all satisfy the majority criterion, they will pick that majority candidate to be the only winner.

(c) A majority candidate has more than half the votes, so everyone else can have at most less than half the votes, and there is no other majority candidate.

(d) A Condorcet candidate X beats everyone in a head-to-head battle. So any other candidate would lose to X in a head-to-head battle, and thus not be a Condorcet candidate.

□