1. In class we found one division resulting from the adjusted winner method for the following problem. Below you will find an alternate division that results from using the adjusted winner method.
   A and B want to share a cake with five homogeneous components, C_1 to C_5. Their valuations are as follows:

	C_1	C_2	C_3	C_4	C_5
A	$1/10$	$5/10$	$1/10$	$2/10$	$1/10$
B	$1/10$	$1/10$	$3/10$	$2/10$	$3/10$

   Answer the following questions.

   (a) Let p be the fraction of component C_4 that A receives. Find the value of p corresponding to the division

$$\begin{aligned} A:&\quad C_2 + C_1 + pC_4, \\ B:&\quad (1-p)C_4 + C_3 + C_5 \end{aligned}$$

$$\begin{aligned} 0.5 + 0.1 + 0.2p &= 0.2(1-p) + 0.3 + 0.3, \\ 0.4p &= 0.2, \\ p &= \frac{0.2}{0.4} = \frac{1}{2}. \end{aligned}$$

   (b) To what threshold value r does this division correspond?

   $r = \frac{0.2}{0.2} = 1.$

   (c) Check that this division is indeed equitable.

   $A: \quad C_2 + C_1 + pC_4 = 0.5 + 0.1 + (\frac{1}{2} \cdot 0.2) = 0.7;$

   $B: \quad (1-p)C_4 + C_3 + C_5 = (1 - \frac{1}{2}) \cdot 0.2 + 0.3 + 0.3 = 0.7.$

   (d) Explain why this division is Pareto-optimal.

   It is a threshold division and all threshold divisions are Pareto-optimal.

2. Suppose A and B divide a cake consisting of 4 homogeneous components, C, S, P, and W. Suppose the valuations are

	C	S	P	W
A	$6/15$	$3/15$	$1/15$	$5/15$
B	$6/15$	$3/15$	$3/15$	$3/15$

   Find 2 *different* divisions using the adjusted winner method and check that each of them is equitable.

   First calculate the A-to-B valuation ratios:

	C	S	P	W
A	$6/15$	$3/15$	$1/15$	$5/15$
B	$6/15$	$3/15$	$3/15$	$3/15$
Ratio	1	1	$1/3$	$5/3$

In decreasing order, the ratios are $5/3$ for W, 1 for C and S, and $1/3$ for P.

The threshold line for the adjusted winner method starts at $r = 1$. The resulting division, if p is the fraction of component C that A receives and q is the fraction of component S that A receives, is

$$\begin{aligned} A: &\quad W + pC + qS, \\ B: &\quad P + (1-p)C + (1-q)S. \end{aligned}$$

Substitute in the valuations and set the share equal:

$$\begin{aligned} \frac{5}{15} + p\frac{6}{15} + q\frac{3}{15} &= \frac{3}{15} + (1-p)\frac{6}{15} + (1-q)\frac{3}{15} \\ \frac{12}{15}p + \frac{6}{15}q &= \frac{7}{15} \\ 12p + 6q &= 7. \end{aligned}$$

If $q = 0$, then $p = \frac{7}{12}$ works, and if $q = 1$, then $p = \frac{1}{12}$ works.

Plug these in to check: for $p = \frac{7}{12}$ and $q = 0$,

$$\begin{aligned} \frac{5}{15} + \frac{7}{12} \cdot \frac{6}{15} &= \frac{5}{15} + \frac{5}{12} \cdot \frac{6}{15} + \frac{3}{15}, \\ \frac{5}{15} + \frac{7}{30} &= \frac{6}{15} + \frac{5}{30}, \\ \frac{17}{30} &= \frac{17}{30}. \end{aligned}$$

For $p = \frac{1}{12}$ and $q = 1$,

$$\begin{aligned} \frac{5}{15} + \frac{1}{12} \cdot \frac{6}{15} + \frac{3}{15} &= \frac{3}{15} + \frac{11}{12} \cdot \frac{6}{15}, \\ \frac{8}{15} + \frac{1}{30} &= \frac{3}{15} + \frac{11}{30} \\ \frac{17}{30} &= \frac{17}{30}. \end{aligned}$$