Homework 20: Chapter 21 Solutions

Other Exercises: (Solutions)

1. In class, we saw the following example:

	Choc	Van	Straw
A	.4	.3	.3
B	.2	.2	.6

And three cuts:

	S_A	S_B
Choc	1	0
Van	.5	.5
Straw	.2	.8

	T_A	T_B
Choc	1	0
Van	.5	.5
Straw	0	1

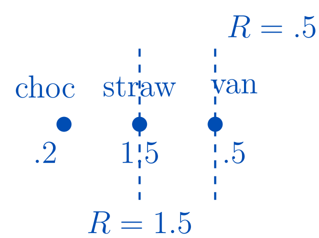
	R_A	R_B
Choc	1	0
Van	.8	.2
Straw	.1	.9

The first cut is not a threshold cut. The second cut is a threshold cut.

(a) Is the third cut a threshold division? Explain why or why not. (You will need to recall the A to B valuation ratios in your argument) A to B valuation ratios are:

	Choc	Van	Straw
A to B ratio	2	3/2	1/2

If we graph the components in search of a threshold division, then the $R_A R_B$-division requires us to draw a threshold line at $R = 3/2$ since the vanilla is divided between them. But we would also have to draw a line at $R = 1/2$ because the strawberry is divided. This does NOT meet the definition of a threshold division, so the $R_a R_B$-division is NOT a threshold division.



(b) By Theorem 21.2, which cuts can we conclude are pareto-optimal? The $T_a T_B$-cut is pareto optimal

(c) By Theorem 21.2, which cuts can we conclude are NOT pareto-optimal? The $S_A S_B$ and the $R_A R_B$ cuts are NOT pareto-optimal

(d) (Optional, harder) For each cut which is NOT pareto-optimal by Theorem 21.2, find an objective improvement. The $S_A S_B$ cut was objectively improved by the $R_A R_B$ cut, as seen in class.

The values the players have for the $R_A R_B$ cut are

	R_A	R_B
A	.67	
B		.58

Since A values Vanilla and Strawberry the same, giving A more Vanilla and less Strawberry will not affect A's value of A's slice, as long as the exchange was even. If we do that, we give B more Strawberry which is very valuable to B. Let's try it and see what happens. Below is the cut, on the right is their values for their slices. We can see that this new cut is an objective improvement to the $R_A R_B$ cut.

	P_1	P_2
Choc	1	0
Van	.9	.1
Straw	0	1

	P_1	P_2
A	.67	
B		.62

2. Solutions in progress. In the meantime, check your math with the Division applet!

   (Extra practice on threshold divisions) Consider the same cake but two new players C and D who have the following valuations:

	Choc	Van	Straw
C	.1	.6	.3
D	.2	.2	.6

   (a) What are the C to D valuation ratios?

   (b) Which of these cuts is a threshold division? Explain.

	S_C	S_D
Choc	1	0
Van	.8	.2
Straw	0	1

	T_C	T_D
Choc	.8	.2
Van	1	0
Straw	.5	.5

   The $T_C T_D$ cut is a threshold cut but the other one is not.

   (c) Applying Theorem 21.2: Which of these cuts is pareto-optimal? Which is not pareto-optimal?

   (d) (Optional, harder) For each cut which is NOT pareto-optimal, find an objective improvement.