## Homework 20: Chapter 21

Recall that on the next quiz, there will be a proof question.
Reading: Chapter 21
Book Exercises: 16.5 (sol), 21.1, 21.2 (Solutions)
Other Exercises: (Solutions)

1. In class, we saw the following example:

|  | Choc | Van | Straw |
| :---: | :---: | :---: | :---: |
| $A$ | .4 | .3 | .3 |
| $B$ | .2 | .2 | .6 |

And three cuts:

| $S_{A} \quad S_{B}$ |  |  | $T_{A} T_{B}$ |  |  |  | $R_{A}$ | $R_{B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Choc | 1 | 0 | Choc | 1 | 0 | Choc | 1 | 0 |
| Van | . 5 | . 5 | Van | . 5 | . 5 | Van | . 8 | . 2 |
| Straw | . 2 | . 8 | Straw | 0 | 1 | Straw | . 1 | . 9 |

The first cut is not a threshold cut. The second cut is a threshold cut.
(a) Is the third cut a threshold division? Explain why or why not. (You will need to recall the $A$ to $B$ valuation ratios in your argument)
(b) By Theorem 21.2, which cuts can we conclude are pareto-optimal?
(c) By Theorem 21.2, which cuts can we conclude are NOT pareto-optimal?
(d) (Optional, harder) For each cut which is NOT pareto-optimal by Theorem 21.2, find an objective improvement.
2. (Optional extra practice on threshold divisions) Consider the same cake but two new players $C$ and $D$ who have the following valuations:

|  | Choc | Van | Straw |
| :---: | :---: | :---: | :---: |
| $C$ | .1 | .6 | .3 |
| $D$ | .2 | .2 | .6 |

(a) What are the $C$ to $D$ valuation ratios?
(b) Which of these cuts is a threshold division? Explain.

|  | $S_{C}$ | $S_{D}$ |  |  | $T_{C}$ | $T_{D}$ |
| :---: | :---: | :---: | :--- | :---: | :---: | :---: |
| Choc | 1 | 0 |  | Choc | .8 | .2 |
| Van | .8 | .2 |  |  |  |  |
|  |  | Van | 1 | 0 |  |  |
| Straw | 0 | 1 |  |  |  |  |

(c) Applying Theorem 21.2: Which of these cuts is pareto-optimal? Which is not pareto-optimal?
(d) (Optional, harder) For each cut which is NOT pareto-optimal, find an objective improvement.

Need more practice? Make up new examples and check your answers with the Division applet. Download the Wolfram CDF player online (free for students) and either download and open the applet by clicking here or go to the applet page linked above with an internet browser other than Chrome.

With the applet, you can activate the $A$ to $B$ ratios option to check your ratios, and input players values and the data for the slices to check their values for the components. Then you can slide the amount in each slice in search of an objective improvement...

