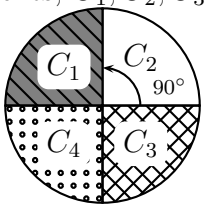
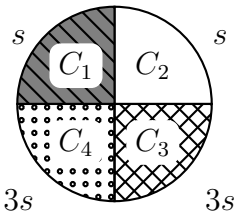
Ch 20 and 17 Additional Problems **Spring 2013**

1. Three cousins Edward (E), Sam (S) and Rebecca (R) have been told they inherited the circular plot of land which is made up of 4 components, C_1, C_2, C_3 and C_4, each composing $1/4$ of the plot.

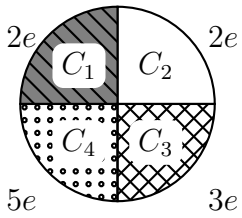


E, S and R decide they will divide up the land between the three of them. Their preferences are as follows:

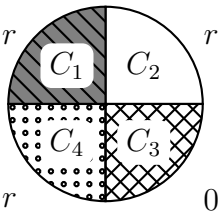
S's preferences:



E's preferences:



R's preferences:



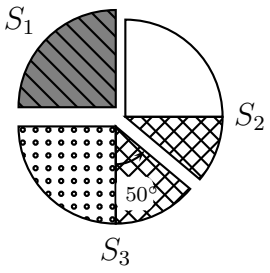
Their valuations of each component are given the table below.

	C_1	C_2	C_3	C_4
E	$2/12$	$2/12$	$3/12$	$5/12$
S	$1/8$	$1/8$	$3/8$	$3/8$
R	$1/3$	$1/3$	0	$1/3$

(a) Suppose first that they decide to use the **lone divider** method to divide up the land. They decide that

- R is the divider, and
- E and S are the choosers.

Suppose R cuts as follows:



i. Fill in the blanks with the fraction of each component that occurs in each slice (this is just a volume measurement):

$S_1 =$_____$C_1 +$_____$C_2 +$_____$C_3 +$_____C_4

$S_2 =$_____$C_1 +$_____$C_2 +$_____$C_3 +$_____C_4

$S_3 =$_____$C_1 +$_____$C_2 +$_____$C_3 +$_____C_4

ii. Fill in the valuations of each slice below using the denominators given:

	S_1	S_2	S_3
E	/18	/18	/18
S	/24	/24	/24
R	/3	/3	/3

iii. List all slices each of S and E think is worth at least $1/3$ of the land in his/her own eyes.

S :_____

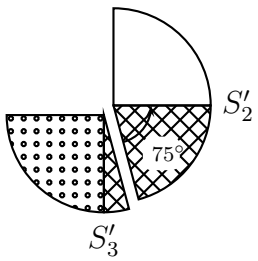
E :_____

iv. List all divisions that could result from using the **lone divider method** where R cuts as shown above and S and E are the choosers.

v. Suppose the outcome is that R gets S_1 and **I cut, you choose** is used to divide the combined slice $S_2 + S_3$ between S and E. They flip a coin for see who divides and it turns out that

- S is the divider, and
- E is the chooser.

S decides to cut as follows:



A. What fraction of the <u>whole</u> cake is the mini cake $S_2 + S_3$ in each of S and E's eyes?

S :_____

E :_____

B. Fill in the blanks with the fraction of each component that occurs in each slice (this is just a volume measurement):

$$S_2' = \_\_\_\_\_ C_1 + \_\_\_\_\_ C_2 + \_\_\_\_\_ C_3 + \_\_\_\_\_ C_4$$
$$S_3' = \_\_\_\_\_ C_1 + \_\_\_\_\_ C_2 + \_\_\_\_\_ C_3 + \_\_\_\_\_ C_4$$

C. Fill in the valuations of each slice as fractions of the mini-cake, $S_2 + S_3$ below using the denominators given:

	S_2'	S_3'
E	$/_{20}$	$/_{20}$
S	$/_{2}$	$/_{2}$

D. List the division that could result from I cut, you choose in this case.

S :_____
E :_____

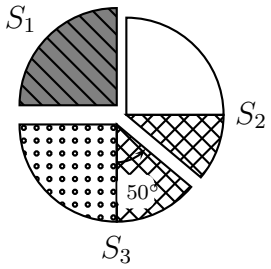
E. Given your result above, list the shares resulting from using the **lone divider method** (so these are the fraction of the cake each person thinks they receive in their own eyes as a fraction of the *whole* cake, $C_1 + C_2 + C_3 + C_4$).

E's share =_____
S's share =_____
R's share =_____

(b) Rebecca recently learned about the method of **Selfridge and Conway** and that it is an envy-free method, which lone divider is not. Therefore, the three now decide to divide the case using the method of **Selfridge and Conway** where

- R is the divider,
- E is the trimmer, and
- S is the chooser.

R cuts the cake in the same way as he did above for the lone divider method giving the same valuations of slices as before.



i. In which of the following ways might E trim (given the valuations of the slices you found earlier). Circle all that apply.

A. Remove $\frac{1}{3}$ of the C_1 part of S_1.
B. Remove $\frac{1}{4}$ of the C_1 part of S_1.
C. Remove $\frac{1}{2}$ of the C_2 part of S_2.
D. Remove all of the C_3 part of S_2.
E. Remove $\frac{1}{2}$ of the C_3 part of S_3.
F. Remove $\frac{2}{3}$ of the C_4 part of S_3.

ii. Suppose E decides to trim $\frac{1}{3}$ of the C_4 part plus all of the C_3 part of S_3. The part of S_3 left after the trimming is called S_3'.

A. Fill in the blanks with the fraction of each component that occurs in each slice (this is just a volume measurement):

$$\text{trimmings} = \_\_\_\_\_ C_1 + \_\_\_\_\_ C_2 + \_\_\_\_\_ C_3 + \_\_\_\_\_ C_4$$
$$S_3' = \_\_\_\_\_ C_1 + \_\_\_\_\_ C_2 + \_\_\_\_\_ C_3 + \_\_\_\_\_ C_4$$

B. Fill in the valuations of each piece below (as fractions of the whole cake):

	S_1	S_2	S_3'	trimmings
E				
S				
R				

C. The method tells us that first they must assign slices, ignoring the trimmings for now, this is Round 1. In what order will E, S and R choose slices in round 1?

(1)_____
(2)_____
(3)_____

iii. Now, they must divide up the trimmings. Suppose that at the end of round 1, S ends up with an untrimmed slice.

- Which of E, S and R should cut the trimmings?

- After the trimmings are cut, in what order to E, S and R choose a piece of the trimmings?

(1)_____
(2)_____
(3)_____

- After both the slices and trimmings have been distributed, explain the following statement:

R does not envy anyone else.