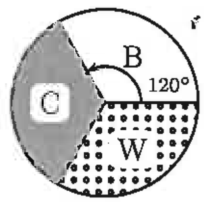
Ch 17 and Equal Division Additional Problems **Spring 2013**

1. Consider the following cake, which is $1/3$ Chocolate (C), $1/3$ Blueberry (B) and $1/3$ Walnut:



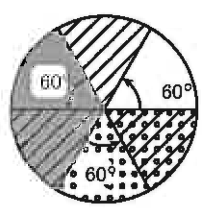
(a) Suppose two people, Peter (P) and Dina (D), wish to share the cake. In your last homework you summarized their preferences as follows:
Valuations of the different components:

	C	B	W
P	$5/7$	0	$2/7$
D	$1/6$	$2/6$	$3/6$

i. Consider the division between P and D shown below, which results from the **equal division** method. (Note: we call a division resulting from doing the equal division method an equal division)

= portion assigned to P

= portion assigned to D



give 1/2 of each flavor to P, same to D.

Verify that

$$(P\text{'s share}) = (D\text{'s share}) = 1/2$$

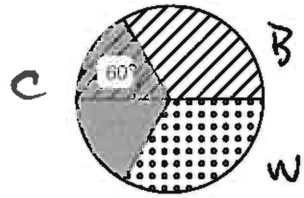
value P's share = $\frac{5}{7} \times \frac{1}{2} + 0 \times \frac{1}{2} + \frac{2}{7} \times \frac{1}{2} = \left(\frac{5}{7} + 0 + \frac{2}{7}\right) \times \frac{1}{2} = 1 \times \frac{1}{2} = \frac{1}{2}$

see why value D's share = 1/2 also?

ii. Notice that in the equal division above both P and D receive half of the cake by volume. Let's now consider an **alternative division** that also assigns P and D half of the cake by volume, shown below.

= portion assigned to P

= portion assigned to D



A. Write down what *fraction* of each component each slice is made up of:

P's slice $= S_1 =$ $\underline{1/2}$ $C +$ $\underline{1}$ $B +$ $\underline{0}$ W
D's slice $= S_2 =$ $\underline{1/2}$ $C +$ $\underline{0}$ $B +$ $\underline{1}$ W

B. Is this division an **equal division** (i.e. does it satisfy the definition of the equal division method)?
Circle One: Yes No

C. Why is assigning each person $1/N$ the volume of the *whole* cake not enough to qualify as an equal division ?
(Hint: Does this tell you what fraction of each *component* a person gets?)
their valuations for the components differ, so by volume does not guarantee equal values.

D. Find each person's share in the **alternative division**.

P: $\underbrace{\frac{1}{2}\times\frac{5}{7}}_{C} + \underbrace{1\times 0}_{B} + \underbrace{0\times\frac{2}{7}}_{W} = \frac{5}{14}$

P's share $=$ $\underline{5/14}$.
D's share $=$ $\underline{7/12}$

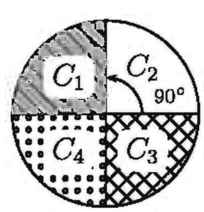
D: $\frac{1}{2}\times\frac{1}{6} + 0\times\frac{1}{3} + 1\times\frac{1}{2} = \frac{1}{12} + \frac{1}{2} = \frac{7}{12}$

E. Is the **alternative division** an objective improvement over the equal division?

No – we gave D more but had to take away from P.

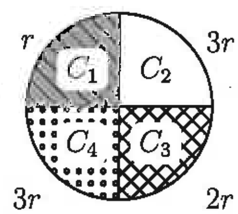
(Remark: we cannot yet conclude the equal arrangement isn't pareto-optimal... even though that is true. Can you see an objective improvement?)

2. Three cousins Edward (E), Sam (S) and Rebecca (R) have also been told they inherited the circular plot of land which is made up of 4 components, C_1, C_2, C_3 and C_4, each composing $1/4$ of the plot.



E, S and R decide they will divide up the land between the three of them. E and S's valuations of the components are given below and R's preferences are as follows:

next page (sorry spacing is weird)



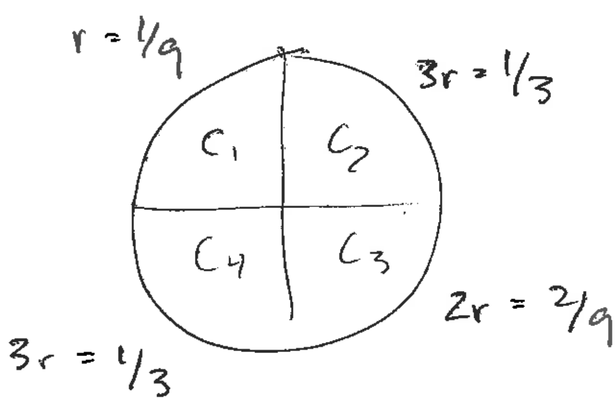
homogeneous land yay!

Solve for r:

$$r + 3r + 2r + 3r = 1$$

$$\iff 9r = 1$$

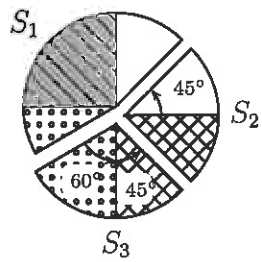
$$\iff r = 1/9$$



Use this to fill in R's valuations of each component in the table below.

	C_1	C_2	C_3	C_4
E	$1/12$	$4/12$	$4/12$	$3/12$
S	$1/7$	$2/7$	$1/7$	$3/7$
R	$1/9$	$1/3$	$2/9$	$1/3$

Suppose E cuts as follows:



The valuations of each slice are as follows:

	S_1	S_2	S_3	Bid list
E	$1/3$	$1/3$	$1/3$	$S_1\ S_2\ S_3$
S	$6/14$	$3/14$	$5/14$	$S_1\ S_3$
R	$7/18$	$5/18$	$6/18$	$S_1\ S_3$

(a) List all slices each of S and R think is worth at least $1/3$ of the land in his/her own eyes.

is $\frac{5}{14} \overset{?}{\geq} \frac{1}{3}$

$\Leftrightarrow 15 \geq 14$ yes!

S: $S_1\ S_3$

R: $S_1\ S_3$

(b) List all divisions that could result from using the **lone divider method** where E cuts as shown above and S and R are the choosers. Indicate whether or not each division is envy-free. If it is not envy-free list all incidences of envy.

1. Give E S_2, give S S_1 and give R S_3.

S thinks R gets $\frac{5}{14} \leq \frac{6}{14}$ ✓ no envy

R thinks S gets $\frac{7}{18} > \frac{6}{18}$ R envies S, not envy-free

(remark: E has no envy and no one envies E)

2. This is not remedied by giving S_2 to E, S_3 to S and S_1 to R.

Now S envies R even though R doesn't envy S.

Bummer. Food for thought - what if we played I cut you choose with S, R and $S_1 + S_3$? This is ~~not~~ the lone divider method... but could we get something envy-free...?

3. Consider the following collection of 12 DVDs consisting of 3 types:

- 2 Romance DVDs (R)
- 4 Horror DVDs (H)
- 6 Comedy DVDs (C)

We will represent the DVDs in the following diagram where one small square represents 1 DVD (all small squares are identical in area):

H	R	R
H	C	C
H	C	C
H	C	C

(a) Which of the following divisions represents a division resulting from the **equal division** method among 2 people, A and B? Circle all that apply.

means give each player half of each type of DVD.

In all options:


= square assigned to A

= square assigned to B