Homework 18: Chapter 17

Reading: Chapter 17

Book Exercises: 17.1, 17.2, 17.3, 17.4 (Solutions)

Other Exercises: (Solutions)

1. Recall the example from class. Three players A, B, C are sharing a Huckleberry, Mulberry, Gooseberry, and Loganberry pie. Their values are given by

	H	M	G	L
A	1/3	0	1/3	1/3
B	1/2	1/4	1/4	0
C	1/4	1/4	1/4	1/4

When C made the following cut on the left, we got the values table on the right:

	S_1	S_2	S_3
H	0	1	0
M	1/3	0	2/3
G	1	0	0
L	0	1/3	2/3

	S_1	S_2	S_3	Bid list
A	1/3	4/9	2/9	S_1, S_2
B	1/3	1/2	1/6	S_1, S_2
C	1/3	1/3	1/3	S_1, S_2, S_3

We can finish Steinhaus' Lone Divider method here because it is possible to allocate the slices fairly. There is no way to create an envy-free division in this example with Steinhaus' Lone Divider method. However, an interesting question is: what if A and B played I cut you choose? Can we get something envy-free? Let's explore what happens if they do.

Note that what follows is NOT going to produce a division which results from Steinhaus' Lone Divider method!!!

(a) A and B are playing I cut you choose with S_1 and S_2 - they let C keep S_3. The total amount of pie to be distributed is all of H, 1/3 of M, all of G, and 1/3 of L. If A makes the following cut, what are the values of the new slices to A and B? (note that in this new mini-envy-table, the sum of the values does not add up to 1! How can we still check our work?)

	S_1'	S_2'
H	1	0
M	0	1/3
G	0	1
L	1/6	1/6

(b) Verify that this is a cut A would make in I cut you choose. Which slice would B pick? Are these slices fair to A and B? Do A or B envy each other?

(c) The new slices are now

	S_1'	S_2'	S_3
H	1	0	0
M	0	1/3	2/3
G	0	1	0
L	1/6	1/6	2/3

For the final divison, give S_3 to C and give slices to A and B according to your work above. Construct the envy table for this division.

i. Do A or B have envy?

ii. Does C have envy?

iii. Is this division an objective improvement to one of the original divisions which resulted from Steinhaus' Lone Divider method? Explain why or why not.

iv. You should hopefully end up observing that this is an objective improvement to one of the divisions, but not to the other (this actually won't necessarily happen, but it does here). Here are some food for thought questions:

- Which division which originally resulted from Steinhaus' Lone Divider method did admit an objective improvement?
- Consider some other example: If you are going to play I cut you choose with two of the divider's slices lumped together, and both players A and B have a higher value for the same slice, which player should make the cut in I cut you choose to have hope for finding an objective improvement?