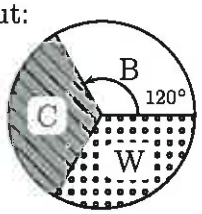
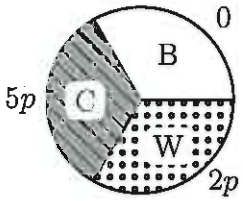
Ch 16 Additional Problems Solutions! Spring 2013

1. Suppose two people, Peter (P) and Dina (D), wish to share the following cake which is 1/3 Chocolate (C), 1/3 Blueberry (B) and 1/3 Walnut:



Suppose their preferences are as follows:

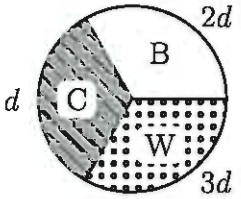
P's Preferences



$5p + 0 + 2p = 1$

$\Leftrightarrow p = 1/7$

D's Preferences



$d + 2d + 3d = 1$

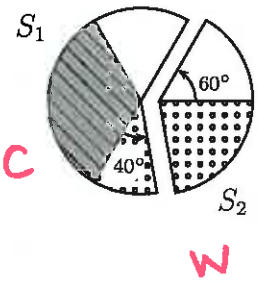
$\Leftrightarrow d = 1/6$

Fill in the following table with their valuations of the different components:

	C	B	W
P	5/7	0	2/7
D	1/6	1/3	1/2

Suppose they decide to use **I cut, you choose** to divide the cake where D is the cutter and P is the chooser. Justify all your answers below.

(a) Verify that D could cut as follows.



D's val $S_1 = \frac{1}{6}\times 1 + \frac{1}{3}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{3}$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

(D's val S_2 is also $= 1/2$)

	S_1	S_2
C	1	0
B	1/2	1/2
W	1/3	2/3

Slices have equal value so yes, D would make this cut

(b) What does P think each slice is worth?

	S_1	S_2
P	17/21	4/21

P's val S_1 =

$\frac{5}{7}\times 1 + 0 + \frac{1}{2}\times\frac{2}{7} + \frac{2}{7}\times\frac{1}{3}$

$= \frac{5}{7} + \frac{2}{21} = \frac{17}{21}$ ✓

(c) List the division that could result from using **I cut, you choose** with the cut given above by listing the slice each of P and D receives in the blanks below.

P: S_1

D: S_2

(d) Is the division envy-free?

yes - it is fair and $N=2 \Rightarrow$ envy-free

(e) Is the division equitable?

no, P thinks P gets $17/21 \neq 1/2$ = D thinks D gets

(f) Is the division Pareto optimal?

no, all of the Blueberry given to P is wasted.
An objective improvement is give D all the Blueberry, because D benefits and P gets the same value from P's slice.

2. Suppose 2 housemates Adam (A) and Bob (B), who are moving out, wish to share a DVD collection of 12 DVDs consisting of 3 types:

   - 2 Romance DVDs (R)
   - 4 Horror DVDs (H)
   - 6 Comedy DVDs (C)

r = # Romance
there are 2r # Horror and 3r # Comedy

We will represent the DVDs in the following diagram where one small square represents 1 DVD (all small squares are identical in area):

H	R	R
H	C	C
H	C	C
H	C	C

A and B's preferences for the different types of DVDs are as follows:

- A likes all 3 types of DVDs equally.
- B likes Romance and Comedy DVDs equally but likes Horror twice as much as he likes either of the others.

A: H R C
$2r + r + 3r = 1$

$\Leftrightarrow \quad r = 1/6$

B: H R C
$\underline{2 \times 2s} + \underline{1 \times s} + \underline{1 \times 3s} = 1$

↑ like H 2x others

↖ ↗ B likes these the same

(a) Fill in the charts below with A and B's preferences given that:

a = the amount that A values 1 Comedy DVD
b = the amount that B values 1 Comedy DVD

A's preferences:

total = _____ a

B's preferences:

total = _____ b

B: $4s + s + 3s = 1$

$\Leftrightarrow s = 1/8$

(b) Fill in the following table with their valuations of the different components:

	H	R	C
A	1/3	1/6	1/2
B	1/2	1/8	3/8

(c) Suppose A and B want to share the DVDs using the method of I cut, you choose where B cuts and A chooses. Answer the following questions.

i. In which of the following ways might B cut? Circle all that apply.

I.

	S_1	S_2
H	3/4	1/4
R	0	1
C	1/3	2/3

II.

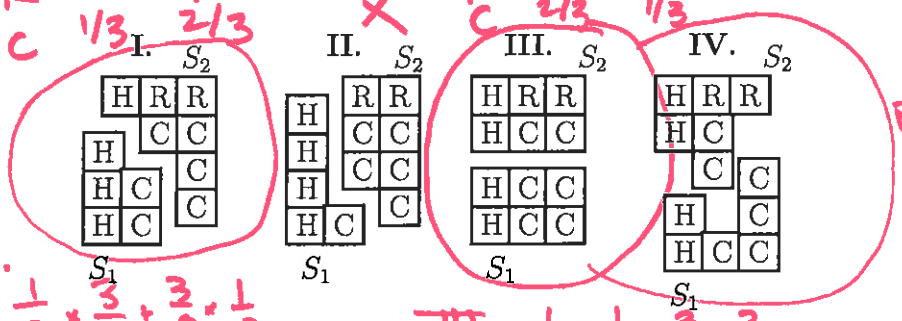
	S_1	S_2
H	1	0
R	0	1
C	1/6	5/6

III.

	S_1	S_2
H	1/2	1/2
R	0	1
C	2/3	1/3

IV.

	S_1	S_2
H	1/2	1/2
R	0	1
C	2/3	1/3



I. $\frac{1}{2} \cdot \frac{3}{4} + \frac{3}{8} \cdot \frac{1}{3}$

$= 3/8 + 1/8 = 1/2$

III. $\frac{1}{2} \cdot \frac{1}{2} + \frac{3}{8} \cdot \frac{2}{3}$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

B's vals S_1

B val S_1: $\frac{1}{2} \cdot \frac{1}{6} + \frac{3}{8} \cdot \frac{1}{6}$

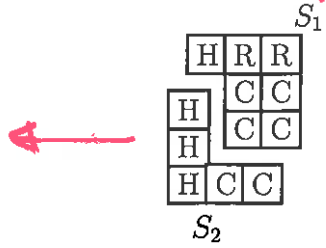
$= \frac{1}{2} + \frac{1}{16}$ NO

$> 1/2$

B val S_1: $\frac{1}{2} \cdot \frac{1}{2} + \frac{3}{8} \cdot \frac{2}{3}$

$= 1/2$ (same as III.)

ii. Suppose B cuts as follows:



	S_1	S_2
H	1/4	3/4
R	1	0
C	2/3	1/3

iii. What does A think each slice is worth?

	S_1	S_2
A	7/12	5/12

Aval S_2 =

$\frac{1}{3} + \frac{3}{4} + \frac{1}{2} + \frac{1}{3} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

iv. List the division that could result from using **I cut, you choose** with the cut given above by listing the slice each of A and B receives in the blanks below. This will be your **original division**.

A : S_1 (A's share) = 7/12

B : S_2 (B's share) = 1/2 ← check that B made a good cut for I cut you choose!

v. Is the division envy-free?

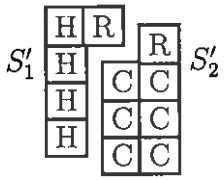
Yes, fair and $N=2 \Rightarrow$ envy-free.

vi. Is the division equitable?

No, got different fractions.

vii. Consider the **alternative division** below.

	S_1'	S_2'
H	1	0
R	1/2	1/2
C	0	1



A gets S_2' and B gets S_1'

A. Fill in the following table with their valuations of the slices in the **alternative division** above:

	S_1'	S_2'
A		7/12
B	9/16	

B. Identify each person's share in this alternative division.

(A's share) =____

(B's share) =____

C. Is this alternative division an objective improvement over the original division?

Circle One: (Yes) No

Please Explain: A gets the same amount and B gets more.

Bonus round: Explain why this new cut is pareto optimal using ch 21 ideas. Can you find an equitable, pareto-opt. division for these players?