1. Suppose two people,Peter $(P)$ and Dina $(D)$, wish to share the following cake which is $1 / 3$ Chocolate (C), $1 / 3$ Blueberry (B) and $1 / 3$ Walnut:


Suppose their preferences are as follows:


Fill in the following table with their valuations of the different components:

|  | $C$ | $B$ | $W$ |
| :--- | :--- | :--- | :--- |
| $P$ |  |  |  |
| $D$ |  |  |  |

Suppose they decide to use I cut, you choose to divide the cake where $D$ is the cutter and $P$ is the chooser. Justify all your answers below.
(a) Verify that $D$ could cut as follows.

(b) What does $P$ think each slice is worth?

|  | $S_{1}$ | $S_{2}$ |
| :--- | :--- | :--- |
| $P$ |  |  |

(c) List the division that could result from using I cut, you choose with the cut given above by listing the slice each of $P$ and $D$ receives in the blanks below.

$$
\begin{aligned}
& P: \\
& D:
\end{aligned}
$$

(d) Is the division envy-free?
(e) Is the division equitable?
(f) Is the division Pareto optimal?
2. Suppose 2 housemates Adam (A) and Bob (B), who are moving out, wish to share a DVD collection of 12 DVDs consisting of 3 types:

- 2 Romance DVDs (R)
- 4 Horror DVDs (H)
- 6 Comedy DVDs (C)

We will represent the DVDs in the following diagram where one small square represents 1 DVD (all small squares are identical in area):

| H | R | R |
| :---: | :---: | :---: |
| H | C | C |
| H | C | C |
| H | C | C |

$A$ and $B$ 's preferences for the different types of DVDs are as follows:

- A likes all 3 types of DVDs equally.
- B likes Romance and Comedy DVDs equally but likes Horror twice as much as he likes either of the others.
(a) Fill in the charts below with $A$ and $B$ 's preferences given that:

$$
\begin{aligned}
a & =\text { the amount that } A \text { values } 1 \text { Comedy DVD } \\
b & =\text { the amount that } B \text { values } 1 \text { Comedy DVD }
\end{aligned}
$$

A's preferences:


$$
\text { total }=\ldots \text { a }
$$

$B$ 's preferences:

|  |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

$$
\text { total }=\ldots \mathrm{b}
$$

(b) Fill in the following table with their valuations of the different components:

|  | $H$ | $R$ | $C$ |
| :--- | :--- | :--- | :--- |
| $A$ |  |  |  |
| $B$ |  |  |  |

(c) Suppose $A$ and $B$ want to share the DVDs using the method of I cut, you choose where $B$ cuts and $A$ chooses. Answer the following questions.
i. In which of the following ways might $B$ cut? Circle all that apply.


II
I. $S_{2}$


| IV. |  | $S_{2}$ |
| :--- | :---: | :---: |
| H R R <br> H C  <br>  C C <br> H  C <br> H C  <br> H C C <br>    |  |  |

ii. Suppose $B$ cuts as follows:

|  | $S_{1}$ |  |
| :--- | :--- | :--- |
| H | R | R |
| H | C | C |
|  | C | C |
| H | C | C |
| H | C | C |
| $\mathrm{S}_{2}$ |  |  |
|  |  |  |

iii. What does $A$ think each slice is worth?

|  | $S_{1}$ | $S_{2}$ |
| :--- | :--- | :--- |
| $A$ |  |  |

iv. List the division that could result from using I cut, you choose with the cut given above by listing the slice each of $A$ and $B$ receives in the blanks below. This will be your original division.

$$
\begin{array}{ll}
A: \quad(A ' \text { 's share })= \\
B:-\quad & (B ' \text { 's share })=
\end{array}
$$

v. Is the division envy-free?
vi. Is the division equitable?
vii. Consider the alternative division below.

A. Fill in the following table with their valuations of the slices in the alternative division above:

|  | $S_{1}^{\prime}$ | $S_{2}^{\prime}$ |
| :--- | :--- | :--- |
| $A$ |  |  |
| $B$ |  |  |

B. Identify each person's share in this alternative division.

$$
\begin{aligned}
& \left(A^{\prime} \text { s share }\right)= \\
& \left(B^{\prime} \text { s share }\right)=
\end{aligned}
$$

C. Is this alternative division an objective improvement over the original division? Circle One: Yes No
Please Explain.

