Homework 15: Chapter 14

1. Suppose there are four bidders A, B, C, and D with bids

$$a = 20 \qquad b = 16 \qquad c = 4 \qquad d = 8$$

and **A is the winning bidder**.

(a) Find q for this compensation arrangement.

(b) What is the equitable compensation arrangement?

(c) Consider <u>instead</u> the following compensation arrangement:

$$B \text{ wins, pays out } x_A = 5,\ x_C = 3,\ x_D = 5.$$

Construct the envy-table for this compensation arrangement. Find all instances of envy.

Solution. (a) Using the formula $q = \frac{w}{S}$,

$$q = \frac{w}{S} = \frac{20}{20+16+4+8} = \frac{5}{5+4+1+2} = \frac{5}{12}$$

(b) The equitable compensation arrangement is determined by what A pays to B, C, and D:

$$x_B = qb = \frac{5}{12} \times 16 = \frac{5 \times 4}{3} = \frac{20}{3}$$
$$x_C = qc = \frac{5}{3}$$
$$x_D = qd = \frac{10}{3}$$

(c) We know what everyone thinks A, C, and D are getting. The only ambiguity is the net gain to B for getting the object.

	A	B	C	D	gets
A	5	7	3	5	
B	5	3	3	5	
C	5	-9	3	5	
D	5	-5	3	5	
thinks					

- A envies B
- B envies A and D
- C envies A and D
- D has no envy.

□

2. Prove: if the winning bidder A is a highest bidder and B, C are the only other bidders, then the compensation arrangement

$$x_B = \frac{a}{3} \qquad x_C = \frac{a}{3}$$

is envy-free.

Proof. First, let's show A doesn't have envy. A thinks everyone else (B and C) gets $a/3$. Actually, A thinks A gets $a/3$ also:

$$x_A = a - \frac{a}{3} - \frac{a}{3} = a - \frac{2a}{3} = \frac{a}{3}$$

so A does not envy anyone.

Also, B and C literally get the same thing, so they don't envy each other. And (for example) B doesn't envy A because $\boxed{b \leq a}$, by assumption that A is a highest bidder, so

$$BtAg = b - \frac{a}{3} - \frac{a}{3} \leq a - \frac{2a}{3} = \frac{a}{3} = BtBg$$

and B doesn't envy A by definition.

The same argument is true for C, so we conclude that neither B nor C has any envy, and the compensation arrangement is envy-free.

□

Remark. Question 2 completes the proof of Our Proposition: An envy-free compensation arrangement is possible if and only if the winning bidder is a highest bidder.