

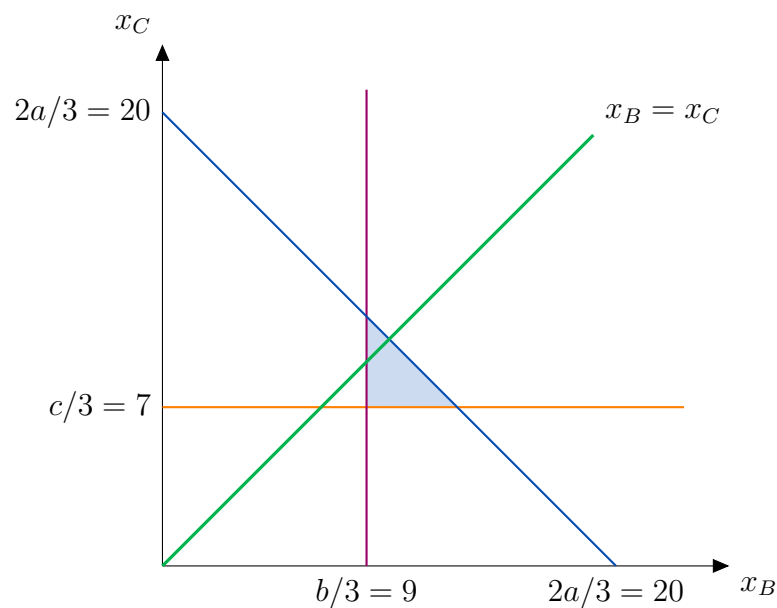
## Homework 12: Chapter 13

**Remark:** Understanding the homework will be directly involved in Thursday's unconventional quiz. Please email or come to office hours if anything is unclear!

**Book Exercises:** 13.5, 13.6 (Solutions)

**Other Exercises:** (Solutions)

- In class, we made the fairness triangle for the example with bids  $a = 30, b = 27, c = 21$  and  $A$  the winning bidder. The fairness triangle in the  $(x_B, x_C)$ -plane is shaded in blue in the picture below. The green line represents all compensation arrangements with equal compensation amounts.



- Remark.** The vertical pink line  $x_B = 9$  represents all compensation arrangements such that  $B$  gets exactly  $B$ 's fair share, which is 9. What does the horizontal orange line represent?
- The slanted blue line is all the compensation arrangements with  $x_B + x_C = 2a/3$ . **Explain:** on this line,  $A$  is getting *exactly*  $A$ 's fair share. Off this line,  $A$  is getting either more or less than  $A$ 's fair share.
- Plot the compensation arrangement

$$x_B = 11 \qquad x_C = 9$$

Is this compensation arrangement going to be fair? Envy-free? Prove your answer.

- What points are associated to the three corners of the fairness triangle?
- Find the intersection point between  $x_B = x_C$  and the line where  $A$  gets  $A$ 's fair share. Plot it in the plane.

2. In class we also studied the situation when  $B$  was the winning bidder. Consider the same bids  $a = 30, b = 27, c = 21$ , and assume  $B$  is a winning bidder. *Paychecks are now going to  $A$  and  $C$ .*
- In the  $(x_A, x_C)$ -plane, graph all compensation arrangements fair to  $A$ .
  - In the  $(x_A, x_C)$ -plane, graph all compensation arrangements fair to  $B$ .
  - In the  $(x_A, x_C)$ -plane, graph all compensation arrangements fair to  $C$ .
  - Draw the fairness triangle. Find the coordinates for the three corners of the triangle.
  - Find the intersection point between  $x_A = x_C$  and the line where  $B$  gets  $B$ 's fair share. Plot it in the plane.
  - Is the compensation arrangement associated to that point fair to
    - A?
    - B?
    - C?
  - Draw the line  $x_A = x_C$ .
  - Can you find an envy-free compensation arrangement for this example? If yes, give one.
3. Consider again three bidders who submit bids  $a = 15, b = 9, c = 21$ .
- What are the fair shares?
  - What is the average bid?
  - Suppose from here on out that  $A$  is the winning bidder. Plot the region of the plane where  $B$  and  $C$  are getting their fair shares.
  - What is the intersection point where  $B$  and  $C$  get *exactly* their fair shares? Call this intersection point  $P$ .
  - Prove:**  $P$  is on the line which represents  $A$  getting *exactly*  $A$ 's fair share.
  - Draw the fairness triangle.
  - Draw the line  $x_B = x_C$ .
  - Do there exist envy-free compensation arrangements for this example?
4. (Optional) Prove: For any number of bidders, the sum of the fair shares is equal to the average bid.