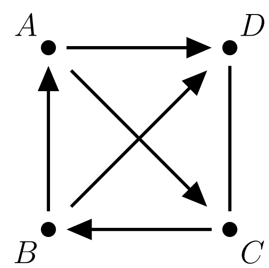
Beatpath method practice

Contents

1 The exercises

1. In the back of the book, there is a remark on Exercise 6.2 that says we can forget about Candidates D, E and F because they do not have any beatpaths going towards A, B or C. "Forget" means ignore them and ALL ARROWS associated to them. D, E, and F are forgettable because they are non-Smith candidates. For Smith candidates, we cannot just forget about any old losers once they have lost, because there might be beatpaths passing through them. See this example:

3	2	3	1	1
A	D	B	C	A
C	C	A	B	C
B	B	D	A	B
D	A	C	D	D



   (a) Check that the unweighted graph provided is correct. Add the margins of victory.

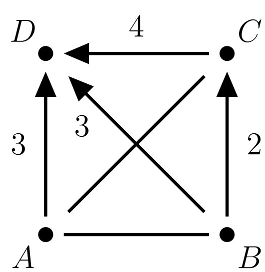
   (b) What is S?

   (c) List all beatpaths between each pair of candidates.

   (d) Who wins by the beatpath method?

   (e) Who wins by pairwise comparison?

2. If the comment in the back of the book on Exercise 6.2 (cited above) was unclear to you, here is a "mini version" where the same phenomenon occurs. Do you see why it is immediately safe to ignore C and D when computing beatpath winners in this particular example?



(a) What is the Smith set?

(b) Explain why we can immediately ignore not only D but also C, even though $C \in S$.

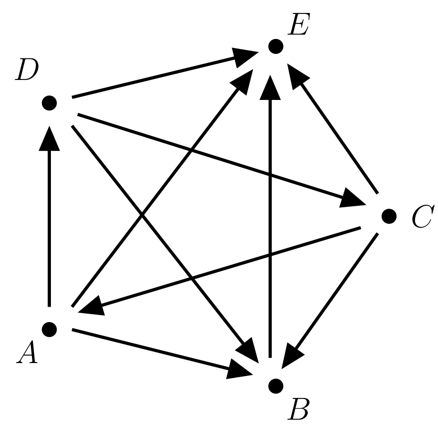
(c) List all beatpaths between pairs of remaining candidates.

(d) Who wins by the beatpath method?

(e) Who wins by pairwise comparison?

3. Consider the following example:

1	2	1	1	2
A	A	C	D	B
D	D	B	E	C
B	C	A	C	E
C	E	E	A	A
E	B	D	B	D



(a) Check that the graph is correct. Fill in the margins of victory.

(b) What is the Smith set?

(c) Who wins by the beatpath method?

(d) Are there weak spoilers for the beatpath method in this example?

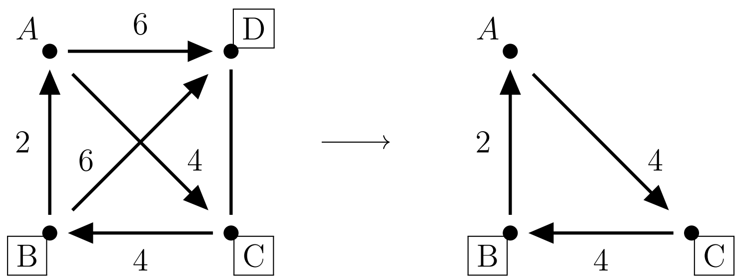
(e) Explain why B cannot be a *losing* spoiler for the beatpath method, without any additional computation.

(f) Check to see if A, D, or C are spoilers in the sense of Chapter 3. Are there winning spoilers? Are there losing spoilers?

(g) Can we conclude that the beatpath method does NOT satisfy the retroactive disqualification criterion?

2 The solutions

1. The weighted comparison graph:



   (a) Check that the unweighted graph provided is correct. Add the margins of victory.

   (b) What is S? $S = \{A, B, C, D\}$

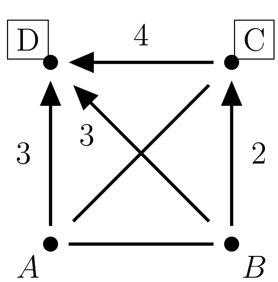
   (c) List all beatpaths between each pair of candidates.

   - There are NO beatpaths AT ALL from D to anyone else. D is a loser and can be circled immediately. We can actually pretend D is completely gone, since there are no beatpaths through D. The graph without D is pictured on the right, if we want it. (Also, notice that D is a Smith candidate, but we get to remove D for a different reason. This works for Non-Smith candidates too, and is part of the reason that the beatpath method is a priori Smith fair.)
   - A to B: $A \xrightarrow{4} C \to B$ strength 4.
     B to A: $B \xrightarrow{2} A$ strength 2. The strongest beatpath from B to A is weaker than the strongest beatpath A to B, so we circle B on the graph as a loser by beatpath.
   - A to C: $A \xrightarrow{4} C$ strength 4.
     C to A: $C \to C \xrightarrow{2} A$ strength 2.
     The strongest beatpath from C to A is weaker than the strongest beatpath A to C, so we circle C on the graph as a loser by beatpath.

   (d) Who wins by the beatpath method? A

   (e) Who wins by pairwise comparison? A and B tie: they both have 2 pairwise comparison points.

2. The second example:



(a) $S = \{A, B, C\}$

(b) Why can we just ignore D? Notice that there are NO beatpaths AT ALL from D to anyone else. Thus, the existence of at least one beatpath to D makes D a loser. We circle D immediately, and then we can forget about D because there cannot be ANY beatpaths that pass through D. (For example, the path $A \to D \to B$ in the graph is NOT a beatpath).

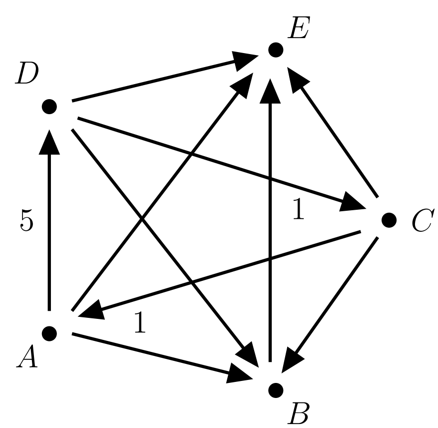
Similarly, C does not outright beat the remaining candidates, A and B. Since B beats C, we can immediately circle C because whatever the strength of that beatpath from B to C is, we know C cannot match it. Now...

(c) We only need to check A and B. There are no beatpaths between them, so NEITHER candidate can be circled as a loser. Therefore, A and B tie by the beatpath method.

(d) B has 2.5 pairwise comparison points, A has 2 pairwise comparison points, C has 1.5 pairwise comparison points, and D has 0. Thus, B wins by pairwise comparison. (interesting! Sometimes pairwise comparison is selective...)

3. The last example:

1	2	1	1	2
A	A	C	D	B
D	D	B	E	C
B	C	A	C	E
C	E	E	A	A
E	B	D	B	D



(a) Check that the graph is correct. Fill in the margins of victory. The only margins of victory we have to worry about are now added.

(b) What is the Smith set? $\{A, C, D\}$

(c) Who wins by the beatpath method? A and C tie.

(d) Are there weak spoilers for the beatpath method in this example? No: The beatpath method is a priori Smith fair, so there are no weak spoilers ever for the beatpath method.

(e) Explain why B cannot be a *losing* spoiler for the beatpath method, without any additional computation. B cannot be a losing spoiler because $B \notin S$, and the beatpath method has no weak spoilers.

(f) Check to see if A, D, or C are spoilers in the sense of Chapter 3. Are there winning spoilers? Are there losing spoilers?

- A could be a winning spoiler. If we remove A, the new winner set for beatpath is $\{D\}$ (there is no longer a beatpath from C to D). The winner should *should* have been just $\{C\}$ and it wasn't, so A is a winning spoiler for beatpath.
- C could be a winning spoiler. Remove C and A beats D so the new winner set is $\{A\}$, which is what should have happened. Then C is NOT a winning spoiler for beatpath.
- D could be a losing spoiler. If we remove D, the new winner set is just $\{C\}$. Since this is different, we conclude D is a losing spoiler.

(g) Can we conclude that the beatpath method does NOT satisfy the retroactive disqualification criterion? The beatpath method fails the retroactive disqualification criterion.